Physics of elliptical reflectors at large reflection and divergence angles I: Their design for nano-photonic integrated circuits and application to low-loss low-crosstalk waveguide crossing

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Abstract

In this work, we study the formulation of beam propagation in an elliptical reflector in nanophotonic integrated circuits (NPICs). We introduce a Gaussian beam mode analysis with Effective Refractive Index Approximation (EIA) applied to planar waveguides. The wavelength-scale waveguides in NPICs lead to large diffraction angle of the input beam and phase aberration in the reflected beam. However, this aberration can be shown to be negligible under first order approximation (FOA). With the Gaussian beam formulation, we study the Near Field and Far Field scenarios in the incident and reflected beams and propose a design methodology that utilizes the two scenarios to minimize footprint of associated devices. Specifically, we use Finite-Difference Time-Domain (FDTD) simulation to show that applying such methodology in designing a waveguide crossing using elliptical reflectors not only minimizes the device footprint but also creates a novel crossing that outperforms the conventional Direct Waveguide Crossing (DWC) in terms of the lowest order mode transmission efficiency and crosstalk.

1. Introduction

In integrated optics, traditional refractive lens cannot be easily integrated on a photonic integrated circuit (PIC). Curved reflectors thus become the most convenient way to perform the beam transformation functions of optical lenses. The typical functions include beam size enlargement or reduction [1–7], phase front transformation between plane phase front and curved phase front [8], beam angular turning [9], optical propagation, and optical cavity applications [10]. In the past, PICs are based on weakly guiding rib waveguides. Curved reflectors are seldom used in weakly-guiding waveguide platform as curved reflectors naturally require deep vertical etching into the chip to form the curved vertical reflecting surface. With the advent of nano-photonics technology based on strongly-guiding waveguide, curved reflectors can be more easily fabricated with other photonic device components as the same process for fabricating strongly guiding waveguides will also be suitable for fabricating the curved reflectors. Recently, curved reflectors have been successfully implemented in ring lasers [10]. Thus, curved reflectors shall become a more commonly used component on chip for nanophotonic integrated circuits (NPIC) in the future.

In nanophotonic integrated circuits, as the waveguide is wavelength-scale, the light beam emitted usually diffracts strongly. Meanwhile, the Rayleigh Range of the emitted light beam is the scale of the waveguide width. This means these light beams cannot be approximated by plane waves. Thus a planar mirror should not be used to perform the beam transformation. By analogy with the geometric optics, the use of elliptical shape curved mirror becomes essential to achieve phase transformation. However, due to the small scale of the waveguide, the physical nature of the light beam becomes notable. Thus even with use of the elliptical mirror, the phase aberration cannot be completely eliminated, but to a much smaller extent compared to using planar mirrors. To study the phase aberration we analyze the beam propagation based on Gaussian beam mode analysis. We show that this aberration is to the first order negligible in the context of Gaussian beam modes. The Gaussian beam analysis also enables us to predict the Gaussian beam waist positions. It turns out that the coupling of a Gaussian beam mode and waveguide mode is maximized at the Gaussian beam waist in terms of minimum phase distortion. This in turn enables us to design devices that can realize functions such as waveguide mode size expanding and waveguide crossing. In our next paper [11] we point out that there is amplitude distortion in the reflected beam.
from an elliptical reflector due to higher order Hermite-Gaussian modes which leads to the transmission loss in the output waveguide. This transmission loss can, however, be eliminated via change of orientation of the reflector, which will be elaborated in the next paper. The main focus of this paper is to discuss the methodology and concerns in designing the waveguide Spot Size Converter (SSC) and the waveguide crossing. This includes determination of the shape of the reflector, the placement of the reflector, as well as the placement of the input and output waveguides. We differentiate the scenario where the reflector is in the near field of the incident or reflected beams, so called Near Field scenario, and the scenario where the reflector is in the far field of the incident or reflected beams, so called Far Field scenario. This is because even the shape of the reflector is kept the same, the input and output waveguides can be placed such that the incident and reflected beams are of the Near Field or Far Field scenarios. Beams of the two scenarios have different diffraction angles and beam waist-to-reflector distances, which in turn results in different geometries and footprints of devices. For the purpose to reduce the footprint of devices, use of the two scenarios should be properly considered as elaborated in this paper.

To verify the analytical formulation, we compare the analytical prediction with the results of Finite-Difference Time-Domain (FDTD) method. It turns out that the analytical prediction is in reasonable agreement with the numerical solution. Thus the analytical Gaussian beam modes analysis method provides us a faster way of designing complex NPICs, which otherwise requires formidable computational capacity and time.

As examples of application, we show designs of Spot Size Converter (SSC) and waveguide crossing using elliptical reflectors. There are many integrated SSCs proposed elsewhere [1–7], which mainly include adiabatic and non-adiabatic structures. Adiabatic structure remains the most commonly used design, which is wavelength insensitive, easy to fabricate and low loss. But it suffers fundamental limitation on its minimum size. Even for the best performance parabolic taper, the length of the taper increases with the square of maximum width of the taper thus cannot be used with large waveguides [1]. Besides, when the taper becomes long, additional loss due to surface roughness (scattering loss) will be evident. Many non-adiabatic structures were therefore proposed to reduce the size [4–7] such as the non-adiabatic planar waveguide tapers [4], photonic crystal SSCs [5], and SSC using antiresonant reflecting optical waveguides [7]. They all exhibited good performance in terms of transmission efficiency and device size. However, they shared a limitation that their designs are wavelength sensitive. Other than that, the design process of non-adiabatic planar waveguide tapers is complex and requires heavy computation [4]; the performance of photonic crystal SSCs is subject to fabrication imperfection because of its fine features. In contrast, SSCs using curved reflectors is wavelength insensitive and have sizes comparable to non-adiabatic tapers because it allows light to diffract freely in the two-dimensional free space. Also because the light propagates in the free space, the transmission efficiency is high owing to avoiding of scattering loss. They are also easy to fabricate and can be formed during the same step of forming waveguide. There are mainly three types of monolithic integrated 90° waveguide crossings [12–20]. The first type utilizes SSCs to reduce diffraction at the intersecting region [12–13]. Similarly to the SSC this type of crossing is either large in size (adiabatic SSC or complex in design and fabrication (non-adiabatic SSC). Another type uses MMI to focus input beam to output waveguide [14]. Since the MMI region needs to be longer than twice the beating length, this type is normally large in size. Besides, MMI is inherently wavelength sensitive which limits the operation bandwidth. The last types use periodic structures such as photonic crystal and sub-wavelength grating to enhance coupling through intersecting waveguide [15,16], which result in miniaturized crossing but is wavelength dependent and is subject to design and fabrication difficulty. In contrast, waveguide crossings using curved reflectors are relatively compact in size, less dependent on wavelength especially for large beam size, easy to be fabricated in the same step as waveguide, and extremely low crosstalk owing to free space propagation of light.

The paper is organized as follows. Section 2 describes the scenarios studied and the problems of interest. Section 3 formulates the beam propagation in the presence of an elliptical reflector and focuses on phase front transformation. The parameters of the elliptical reflector are derived under the condition that the phase front transformation requirement is satisfied. Section 4 studies the application of elliptical reflectors to form waveguide crossings. The FDTD simulation shows that the waveguide crossing using elliptical reflector has advantages over the direct waveguide crossing in terms of substantially higher transmission efficiency and lower crosstalk.

2. Gaussian beam mode analysis

2.1. Definition of reflector structure

For the purpose of discussion, we assume that the NPIC is the typical silicon-on-insulator (SOI) platform used in silicon photonics in which a thin (typically 0.2–0.3 μm thick) layer of silicon on top of a thick oxide layer is used to form the strongly guiding planar waveguide that gives strong optical mode confinement vertically. A strongly guiding channel waveguide can be subsequently formed by vertical etching resulting in center high-refractive-index silicon waveguide core surrounded by low refractive index claddings on the side in the horizontal direction. The cladding can be air by simply etching away the silicon or can be filled with a transparent oxide or polymer. This then gives a strong optical mode confinement horizontally. Fig. 1b shows the cross-section of such a SOI waveguide structure. The same technique can

![Fig. 1. SOI waveguide structure (a) top view and (b) cross section view. The SOI waveguide has a Si core (refractive index ncore) and SiO2 cladding (refractive index nclad). The width of the waveguide is about l/(2n0) ~ 0.2 μm such that the waveguide mode is single mode (shown in dashed line), l is the wavelength of the propagating light in vacuum, and n0 is the effective planar propagating refractive index of the waveguide core. The thickness of the waveguide is typically around 0.25 μm.](image)
be used to create a reflector. The reflection occurs at the interface between vertical guided area and the cladding. Additionally, metal coating can be added to the interface so as to increase the reflectivity. The back of the reflector is filled with air or oxide/polymer that has a refractive index of $n_b=1$ to around $n_b=1.5$, which has a high refractive index contrast with the front side of the reflector that is silicon with a refractive index of $n_f=3.5$. This high reflective index contrast can be utilized to achieve strong reflection via total internal reflection that occurs at a large enough incident angle without the use of metal coating. Such metal-coating free reflector has advantage when coming to device fabrication because it reduces the fabrication steps and avoids additional optical loss due to metal residue on the waveguide surface. For example, the critical angle for refractive index combination of $n_f=3.64$ and $n_b=1$ is $\theta_c=16.0^\circ$. Hence in order that the reflector can make use of total internal reflection, the divergence (diffraction) angle of the incident beam should not be too large. Otherwise when the incident angle is less than the critical angle, some of the beam energy at the beam edge with large incident angle will leak out.

As a simplification, we approximate the three-dimensional optical waveguide structures by two-dimensional optical waveguide structures using the effective refractive index method. The details of the effective refractive index method can be found in [21]. Fig. 1a shows the two-dimensional structure of the above-mentioned SOI waveguide. It is worth noting that the effective refractive index method is well known to be a reasonable approximation to reduce 3D waveguide problem to 2D waveguide problem and is widely used in design and simulation of PIC structures such as waveguide tapers and waveguide crossing [9,12–14,18]. This is because the guided light beam does not diffract in the vertical direction. As [20] pointed out, although there are discrepancies between 2D FDTD and 3D FDTD simulation results, they both predict the same device performance. In our application, the most important thing is to match the mode at the channel waveguide mouth. Once it is done, the behavior of light beam in the planar waveguiding region is accurately described by the 2D method. We have inserted a comment in [25] in this regards.

Fig. 2 shows the geometry of a two-dimensional elliptical reflector configuration after applying the effective refractive index method. Light propagates in the $z$–$x$ plane where vertical confinement is provided by the refractive index structure in the vertical direction. Whereas the horizontal confinement is provided by optical elements such as waveguides and reflectors. The polarization of the light can be either TM or TE. For example, the TM polarization is the polarization with the magnetic field vector $\vec{H}$ parallel to the $z$–$x$ plane or the electric field vector $\vec{E}$ pointing in the $y$ direction with magnitude denoted by $E_y$. Thus, for TM case, the representative field will be $E_y$; for TE case with magnetic field pointing in the $y$ direction, it will be $H_y$. Once we know $E_y$ for TM case, the magnetic field in the $z$–$x$ plane can be obtained using Maxwell equations. Similarly for TE case. Throughout this paper, we treat the field as scalar field, $H_y$ (TE) or $E_y$ (TM).

The light beam propagates in channel waveguides as waveguide modes in Fig. 2. The waveguide modes can be solved analytically or numerically using commercial programs given structure parameters such as the waveguide width $D_{w,\text{inc}}$, effective refractive index of waveguide core $n_{\text{core}}$, and cladding $n_{\text{clad}}$ and wavelength $\lambda$. The light beam is free to propagate in the area outside the waveguide as though in two-dimensional free space. Such “free space” modes can be solved analytically using Maxwell equations. In the two-dimensional scenario, under paraxial approximation, the free propagating modes can be described in terms of complete sets of two-dimensional Hermite–Gaussian (HG) beam modes.

In Fig. 2a beam (Gaussian beam mode) propagating in the free space (emitted from an input waveguide) is reflected by an curved reflector (in this case, elliptical) and transformed into another beam (and subsequently coupled into an output waveguide). The reason why the reflector should be elliptical can be explained by analogy with geometrical optics. It is well known that the phase fronts of a Gaussian beam and higher order modes are circular (spherical in three-dimensional case). Thus at certain position, the phase front of the Gaussian beam can be seen as related to phase front of a point source. In geometrical optics, the beam originating from one focus of an ellipse shall be reflected and converged to the other focus of the ellipse. This is because an elliptical surface has the property that any two lines that connect the two focal points and a point on the ellipse have the same total length. Although it is not accurate, as we will elaborate later in this paper, we tentatively assume that the center of the phase front of the beam within the region of the reflector is fixed. We now assume the phase front of the incident beam has its center of curvature at $O_1$. Let the propagation axis of the incident beam to be $O_1C$ where $C$ is on the reflector. The phase front of the reflected beam shall have its center of curvature at $O_2$. The propagating axis of the reflected beam is then defined by $O_2C$. The angle between the propagating axes of the incident and reflected beams are defined as turning angle $\theta$. $(x, y, z)$ coordinate axes are defined such that the $z$-axis lies along $O_1C$, with $z=0$ at $C$ and $z=-R_{\text{in}}$ at $O_1$ (with $z<0$), where $R_{\text{in}}$ is the radius of curvature of phase front of the incident beam at $C$. Similarly, the $(x', y', z')$ coordinate axes are defined so that $z'$ axis coincides with the optical axis of the reflected beam, $O_2C$, with $z'=0$ at $C$ and $z'=R_{\text{out}}$ at $O_2$ (with $z'>0$), where $R_{\text{out}}$ is the radius of curvature of phase front of the incident beam at $C$. In the following context we use subscript $i$ for
variables associated with the input/incident side, subscript out for variables associated with the output/reflected side. The $(z',x')$ plane lies in the plane of the paper in such a way that the $y'$ and $y$ axes coincide. The beam waist radii of the incident and reflected beam are $w_{in}$ and $w_{out}$. It is worth noting that the beam waists of the incident and reflected beams are not at $O_1$ and $O_2$ but at $B_1$ and $B_2$, which coincide with the input and output waveguides. This is because that, due to the requirement of phase-front matching, the phase fronts at the waist of incident and reflected beams should be flat as the phase front of the waveguide mode. Then let the distance from $B_1$ to $C$ be $l_{in}$ and distance from $B_2$ to $C$ be $l_{out}$. The distance between $O_1$ and $B_1$ (or $O_2$ and $B_2$) is sometimes called Physical Optics Correction $\Delta\psi$ (or $\Delta\psi_0$) [23]. The input and output waveguides associated with the incident and reflected Gaussian beams are illustrated in Fig. 2 in dashed lines. The input waveguide width is $D_{WG,in}$ and the output waveguide width is $D_{WG,out}$.

### 2.2 Gaussian beam mode formulation

In this paper, we focus on studying the use of elliptical reflector to transform Gaussian beam sizes. The reason is two-fold: first, as we will show in our next paper [11] that, a specific waveguide can be related to a certain Gaussian beam in terms of Gaussian beam decomposition. Second, transformation of the beam sizes could potentially be applied to realize many other applications, such as waveguide crossings. As far as design is concerned, the problem for us to solve can be stated as, given the input and output waveguide width $D_{WG,in}$ and $D_{WG,out}$ as well as the turning angle $\theta$ (the angle between the input and output waveguides in Fig. 2), determine the shape and position of an elliptical reflector such that the coupling efficiency of light beam from the input waveguide to the output waveguide is maximized. In above statements, by saying “shape”, it implies knowing the major diameter $D_{maj}$ and minor diameter $D_{min}$ of an ellipse; by saying “position”, it implies knowing the angles between the waveguides and axes of the ellipse: $\theta_{in}$, $\theta_{out}$, and the distances between the waveguide axes and $C$: $l_{in}$, $l_{out}$. It turns out that $D_{maj}$, $D_{min}$ and $\theta_{in}$, $\theta_{out}$ are determined by $l_{in}$, $l_{out}$ and $\theta$, $l_{in}$, $l_{out}$ are determined by $w_{in}$, $w_{out}$, $\theta_{in}$, $\theta_{out}$ and $w_{in}$, $w_{out}$. Whereas, as will be elaborated in our next paper [11], the incident and reflected beam waist radii $w_{in}$ and $w_{out}$ are determined by the structure of the waveguide. Since it is beyond the scope of this paper, we assume $w_{in}$ and $w_{out}$ are given a priori. In sum, to obtain solutions to the above problems, we need to find the relationship of $w_{in}$, $w_{out}$, $l_{in}$, $l_{out}$, $\theta_{in}$, $l_{out}$ and $\theta$. These parameters can be related by Gaussian-beam transformation formula, which is discussed in detail in the following sections.

On describing the incident and reflected beams in terms of modal expansions we can write the complex amplitude

$$F(x,Z)=\sum_n c_n \psi_n(x,Z)=\sum_n c_n A_n(x,w(Z)) e^{i\phi_n(x,Z)},$$

where $F(x,Z)$ represents the field $H_0(x,Z)$ for TE mode and $E_0(x,Z)$ for TM mode. $F(x,Z)$ is decomposed in terms of a complete set of Hermite–Gaussian modes represented by $\psi_n(x,Z)$. $\psi_n(x,Z)$ is normalized complex amplitude of two-dimensional Hermite–Gaussian modes of amplitude $A_n(x,w(Z))$ and phase $\phi_n(x,Z)$. $c_n$ is normalized complex mode coefficient. Note that we have let $Z=z-z_0$, where $z_0$ is the beam waist position of the Hermite–Gaussian modes, $x$-axis is the transverse direction to the propagation axis ($z$-axis). $A_n(x,w(Z))$ is given by

$$A_n(x,w(Z))=\sqrt{\frac{2}{\pi w^2}} \frac{1}{2^n} H_n \left(\frac{x}{w(Z)}\right) \exp\left(-\frac{x^2}{w^2(Z)}\right),$$

where $H_n(s)$ represents a Hermite polynomial of order $n$ with argument $s$, $w(Z)$ is the beam radius of the mode at $Z$

$$w(Z)=w_0 \sqrt{1+[Zl/(\pi w_0^2)]^2},$$

where $w_0$ is the beam radius of the Hermite–Gaussian modes. $\phi_n(x,Z)$ is given by

$$\phi_n(x,Z)=\exp\left[-jk \left(\frac{Z^2}{2R(Z)}\right) +j(n+1/2)\Delta\psi_0(Z)\right],$$

where $\Delta\psi_0(Z)$ is the phase slippage for the mode, $R(Z)$ is the radius of curvature of the phase front at $Z$. $\Delta\psi(Z)$ and $R(Z)$ are given by

$$\Delta\psi(Z)=\zeta(Z)=\tan^{-1}[\frac{Zl}{(\pi w_0^2)}],$$

$$R(Z)=Z[1+(\pi w_0^2)/(\zeta^2 Z)],$$

where $\zeta(Z)$ is the Guoy phase term of the mode. Notice that the initial phase of the $n$-th order mode is set to be zero since it can be absorbed in the complex mode coefficient. We also define the Rayleigh Range of a Gaussian-beam as

$$Z_R=\pi w_0^2/\lambda.$$  

Note that $\lambda$ is the wavelength of the beam in the medium.

Before proceeding, we introduce the concept of Physical Optics Correction, Near Field, and Far Field. The Physical Optics Correction for a Gaussian beam is defined as the offset between the center of curvature of the phase front and the beam waist. For the purpose of discussion, we illustrate the caustic of a Gaussian beam defined by the beam radius in Fig. 3. We have assumed beam waist at $Z=0$. At $Z$, the radius of curvature of phase-front is given by (6). Thus we write the Physical Optics Correction at $Z$ as

$$\Delta=R(Z)-Z=Z^2/\pi^2 \lambda Z.$$  

The Physical Optics Correction is always positive, indicating that the center of curvature of phase-front at $Z>0$ is always in region $Z<0$. Also shown in Fig. 3 is the diffraction angle of the beam at two different positions. The diffraction angle at $Z$ is defined by the tangent of the beam radius

$$\tan\theta=\frac{w_0}{2Z^{1/2}w(Z)}=\frac{w_0}{\sqrt{1+(Z/Z_R)^2 Z^2_z}},$$

where we show that at the two different phase front positions $Z_1$ and $Z_2$, the centers of curvature of the phase-front are at different positions $Z=-\Delta_1$ and $Z=-\Delta_2$, where $\Delta_1$ and $\Delta_2$ are given by (8). Besides, the diffraction angles are different at different $Z$ positions according to (9).

We define the region in a Gaussian-beam where $Z<Z_R$ to be the Near Field Region and the region where $Z>Z_R$ to be the Far Field.
Field Region. A property of the Gaussian beam is that, there may be two on-axis positions (when we only consider $Z=0$) that have the same values of the radius of curvature of the phase front. The two positions are said to be in the Near Field region and the Far Field region of the Gaussian beam, respectively. It turns out that, in many applications, a main consideration to address is whether to place the reflector at the Near or Far Field of the incident beam, as well as whether to place the output waveguide at the Near or Far Field from the reflector.

### 3. Reflector design

A complete design of an elliptical reflector includes determination of the shape of the reflector, as well as the positions and sizes of the beam waists of the incident and reflected beams. In this section, we show that the shape of an elliptical reflector is determined by $R_{in}=O_1 C$, $R_{out}=O_2 C$ and $\theta$. The beam waist positions are determined by knowing $L_{in}$ and $L_{out}$. It turns out that $R_{in}$, $R_{out}$, $L_{in}$, $L_{out}$, $w_{in}$ and $w_{out}$ are not independent but related through (3) and (6). Since we are interested in designing a reflector when $w_{in}$ and $w_{out}$ are given, we focus on studying the relationships of $R_{in}$, $R_{out}$, $L_{in}$, $L_{out}$ Given $w_{in}$, $w_{out}$. We also demonstrate how to use the relationships to design an elliptical reflector.

First, we show that the shape of an elliptical reflector is determined by $R_{in}$, $R_{out}$ and $\theta$. To work properly, the shape of the elliptical reflector should be such that it transforms the phase fronts of the incident beam to that of the reflected beam. We have assumed that the centers of phase front of the incident beam and the reflected beam at $C$ are at the foci of the elliptical reflector $O_1$ and $O_2$, respectively. Specifically speaking, at $C$ we have $O_1 C = R_{in}$, $O_2 C = R_{out}$, where $R_{in}$ is the radius of curvature of phase front of the incident beam at $C$, $R_{out}$ is the radius of curvature of phase front of the reflected beam at $C$. Using (6), we can write

$$R_{in} = L_{in}[1 + (\pi w_{in}^2 / \lambda L_{in})^2],$$

$$R_{out} = L_{out}[1 + (\pi w_{out}^2 / \lambda L_{out})^2].$$

This is accurate when both the incident beam and reflected beam are point source beams. However, it is not accurate when the incident and reflected beams are Gaussian beams in that even though the phase front of a Gaussian beam is approximately circular, the advancement of phase front is not linear as in a point source beam. This is illustrated in Fig. 3: the center of curvature of the phase front of a Gaussian beam is not fixed but varies for phase fronts at different positions. The actual center of curvature of a phase front is determined by the Physical Optics Correction. Thus, if the elliptical reflector is designed to correctly transform the phase front at the center of the mirror, it does not transform the phase fronts at other positions on the mirror correctly because the centers of curvature of these phase fronts is not on the foci of the elliptical reflector. This means that in the region occupied by the reflector, there is phase front mismatch between the incident/reflected beam and the corresponding point source beams centered at the foci. However, it has been shown that despite the presence of phase mismatch in the incident beam and reflected beam respectively, the net phase mismatch is negligible outside the region occupied by the reflector [22]. Simply speaking, the phase lead and lag in the incident and reflected beams are compensated by the geometry of the reflector. In Appendix we show this is, to the first order, true through a derivation based on 2D Gaussian beam analysis, which is slightly different from that in [22] which is based on 3D Gaussian beam analysis.

Because the phase mismatch is negligible, we can treat the phase transformation as though the centers of curvature of the phase front of the incident beam and reflected beam are at $O_1$ and $O_2$, respectively. According to Geometry, an ellipse is determined by its major diameter $D_{maj}$, defined as the longest distance between antipodal points on an ellipse, and minor diameter $D_{min}$, defined as shortest distance between antipodal points on an ellipse. We define the major radius $a$ as half of the major diameter; the minor radius $b$ as half of the minor diameter. In Fig. 2, it can be seen that by knowing the lengths of $O_1 C$ and $O_2 C$ and the angle $\angle O_1 C O_2 = \theta$, $a$ and $b$ can be solved. In solving $a$ and $b$ we shall use the fact that the sum of the distances from any point on an ellipse to the two foci of the ellipse is constant and is equal to the major diameter.

$$a = D_{maj}/2 = (O_1 C + O_2 C)/2 = (R_{in} + R_{out})/2. \tag{12}$$

Given the angle $\angle O_1 C O_2 = \theta$, half of the focal length of an ellipse (the focal length is defined as the distance between the two foci) $c$ can be written as

$$c = O_1 O_2/2 = \sqrt{O_1 C^2 + O_2 C^2 - 2O_1 C O_2 \cos \theta}/2 = \sqrt{R_{in}^2 + R_{out}^2 - 2R_{in} R_{out} \cos \theta}/2. \tag{13}$$

Then using the trigonometry in an ellipse, the minor radius is given by

$$b = \sqrt{a^2 - c^2}. \tag{14}$$

In sum, as long as $\theta$, $R_{in}$ and $R_{out}$ are given, the shape of the elliptical reflector is uniquely determined.

Now we derive the relationships of $R_{in}$, $R_{out}$, $L_{in}$, $L_{out}$ under the condition that $w_{in}$ and $w_{out}$ are prescribed. Particularly, we introduce the Near Field and Far Field scenarios when discussing the relationships. The beams of the two scenarios show different properties and can be used in different applications as will be discussed in the next section.

#### 3.1. Near Field and Far Field scenarios of the incident beam

The relationships of $R_{in}$, $L_{in}$ and $w_{in}$ are determined by (3) and (6), which are solved as

$$L_{in}^{near} = R_{in} - \sqrt{-4(\pi w_{in}^2 / \lambda)^2 + R_{in}^2}/2. \tag{15}$$

$$L_{in}^{far} = R_{in} + \sqrt{-4(\pi w_{in}^2 / \lambda)^2 + R_{in}^2}/2. \tag{16}$$

The subscript near and far denote the Near Field scenario and Far Field scenario, which indicates the corresponding $L_{in}$ is in the Near Field or Far Field of the incident beam. This is because in (15), $L_{in}^{near} < \pi w_{in}^2 / \lambda$, and in (16), $L_{in}^{far} \geq \pi w_{in}^2 / \lambda$. For the purpose of generality, we introduce dimensionless quantities $z_{R,in} = R_{in} / (\pi w_{in}^2 / \lambda)$ and $\beta_{L,in} = L_{in} / (\pi w_{in}^2 / \lambda)$ for the incident beam, where $z_{R,in}$ is the relative radius of curvature of phase front of the incident beam normalized by the Rayleigh Range of the incident beam, $\beta_{L,in}$ is the relative on-axis distance of the phase front normalized by the Rayleigh Range of the incident beam. Now (15) and (16) can be rewritten as

$$\beta_{L,in}^{near} = [z_{R,in} - \sqrt{-4 + z_{R,in}^2}] / 2, \tag{17}$$

$$\beta_{L,in}^{far} = [z_{R,in} + \sqrt{-4 + z_{R,in}^2}] / 2. \tag{18}$$

Fig. 4a plots $\beta_{L,in}$ against $z_{R,in}$. The lower branch corresponds to (17). The upper branch of the curve corresponds to (18). The two branches merge at $\beta_{L,in}^{near} = 1$, $z_{R,in} = 2$. Now we can use the dimensionless quantity to define the Near Field scenario as $\beta_{L,in}^{near} < 1$, and the Far Field scenario as $\beta_{L,in}^{far} > 1$. Obviously, given a $z_{R,in}$ value, there are two possible $\beta_{L,in}$ values on the curve. Design-wise, this means that for a given input waveguide (thus the beam waist
radius \( w_{\text{in}} \) is given), and a given reflector shape (implying \( z_{\text{ref}} \) is prescribed), the input waveguide can have two possible positions (indicated by two \( \beta_{\text{in}} \) values), that meet the phase front transformation requirement. These two positions correspond to the Near Field and Far Field solutions of the incident beam at (15) and (16). Besides, by using (3) we know the two configurations have different incident beam radii at \( C \). Given the incident beam radius at \( C \), determination of the reflected beam parameters is derived as follows.

3.2. Near Field and Far Field scenarios of the reflected beam

The reflected beam from an elliptical reflector can be seen as a focused beam that has an initial beam radius \( w_{\text{out}}(C) \) at \( C \) which is determined by the incident beam and the incident reflector shape. As will be elaborated in our next paper [11], it is a sensible choice by letting \( w_{\text{in}} = w_{\text{out}}(C) = w_{C} \).

Then, by solving (3) and (6), we can write the relationship of \( L_{\text{out}} \) and \( R_{\text{out}} \) as

\[
L_{\text{out}} = \frac{R_{\text{out}}}{1 + \frac{R_{\text{out}}}{(\pi w_{C}^{2} / \lambda)^{2}}}. \tag{20}
\]

On the other hand, given \( L_{\text{out}} \), the reflected beam waist radius \( w_{\text{out}} \) is written as

\[
w_{\text{out}} = w_{C} \sqrt{1 + (\pi w_{C}^{2} / L_{\text{out}})^{2}}. \tag{21}
\]

Plugging (20) in (21), we can write \( w_{\text{out}} \) in terms of \( R_{\text{out}} \)

\[
w_{\text{out}} = \sqrt{1 + (\pi w_{C}^{2} / R_{\text{out}})^{2}}. \tag{22}
\]

By introducing dimensionless quantities, \( \gamma_{\text{R, out}} = R_{\text{out}} / (\pi w_{C}^{2} / \lambda) \) and \( \eta_{\text{R, out}} = w_{\text{out}} / w_{C} \), we have the dimensionless form of above equations

\[
\eta_{\text{R, out}} = \frac{\gamma_{\text{R, out}}}{1 + \gamma_{\text{R, out}}}, \tag{23}
\]

\[
\omega_{\text{R, out}} = \frac{1}{\sqrt{1 + \gamma_{\text{R, out}}^{2}}}. \tag{24}
\]

Fig. 4b plots \( \eta_{\text{R, out}} \) against \( \gamma_{\text{R, out}} \). Fig. 4c plots \( \omega_{\text{R, out}} \) against \( \gamma_{\text{R, out}} \). From (24) we know that when \( \gamma_{\text{R, out}} = 1 \), \( \omega_{\text{R, out}} = w_{\text{out}} / w_{C} = 1 / \sqrt{2} \). Thus the Near Field scenario of the reflected beam can be defined by \( \gamma_{\text{R, out}} < 1 \) and the Far Field scenario of the reflected beam is defined by \( \gamma_{\text{R, out}} > 1 \). The Near Field scenario of the reflected beam is: 

\[
\eta_{\text{out}} = \frac{\gamma_{\text{R, out}}}{1 + \gamma_{\text{R, out}}}, \tag{25}
\]

\[
\omega_{\text{out}} = \eta_{\text{out}} / \sqrt{1 + \gamma_{\text{R, out}}^{2}}. \tag{26}
\]

By introducing dimensionless quantities \( \beta_{\text{L, in}} = L_{\text{in}} / (\pi w_{\text{in}}^{2} / \lambda) \), \( \beta_{\text{L, out}} = L_{\text{out}} / (\pi w_{\text{in}}^{2} / \lambda) \), and \( \chi = w_{\text{out}} / w_{\text{in}} \), \( \beta_{\text{L, in}} \) and \( \beta_{\text{L, out}} \) are normalized on-axis distance between the beam waists and \( C \) and \( \chi \) is the normalized reflected beam radius, we write (27) in the dimensionless form

\[
\beta_{\text{L, out}} = \sqrt{\beta_{\text{L, in}}^{2} - (\chi^{2} - 1)/\chi}. \tag{28}
\]

Fig. 5 plots \( \beta_{\text{L, out}} \) against \( \beta_{\text{L, in}} \). In the figure, we mark the positions where \( \beta_{\text{L, out}} = 1 \) and \( \beta_{\text{L, in}} = 1 \) using long dashed lines. The axes are then divided into four regions. Each region corresponds to one combination of the Near Field and Far Field scenarios of the incident and reflected beams. (28) requires that

\[
\beta_{\text{L, in}} > \chi^{2} - 1. \tag{29}
\]

Eq. (29) implies that if \( \chi > \sqrt{2} \), \( \chi^{2} - 1 \) is greater than 1 thus \( \beta_{\text{L, in}} \) is greater than 1, meaning the incident beam can only be the Far Field scenario. Whereas if \( 1 \leq \chi < \sqrt{2} \), \( \chi^{2} - 1 \) is less than 1 thus \( \beta_{\text{L, in}} \) can be less than 1, meaning the incident beam can be both the Far Field and Near Field scenarios. Besides, as long as \( \chi > 1 \), \( \beta_{\text{L, out}} \) value ranges from zero to infinity, meaning the reflected beam can be the Near Field or Far Field scenarios. For \( \chi < 1 \), it can be seen as a situation where the input and output side of reflector is switched. Then, by letting \( \chi = 1/\chi \) we can have the same conclusion as above by just placing \( \chi \) with \( \chi \) and switching \( \beta_{\text{L, in}} \) and \( \beta_{\text{L, out}} \). We show four exemplary curves. For \( \chi = 2.5 \), we have \( \beta_{\text{L, in}} > 1 \), \( \beta_{\text{L, out}} > 0 \). The
incident beam can only be Far Field scenario, while the reflected beam can be Near Field or Far Field scenarios. For $\chi = 1$, the incident beam and reflected beam are identical and can be either Near Field or Far Field scenarios. For $\chi = 0.4$, the incident beam can either Near Field or Far Field scenarios, while the reflected beam can only be the Far Field scenario. It can be seen as the reversed version of $\chi = 2.5$.

4. Application

In this section we introduce two applications of the elliptical reflector. One is to use the elliptical reflector as a SSC; the other is to use two pairs of elliptical reflectors to form a waveguide crossing. We show that in order to minimize the footprint of the devices, for the former application, one should design the reflector such that the reflected beam is Near Field scenario. Whereas for the latter, counter intuitively, one needs to design the reflector such that the reflected beam is the Far Field scenario.

It is worth mentioning that, the SSC and waveguide crossing using elliptical reflectors have advantages over their conventional counterparts. One important property of the elliptical reflector is that the light beam propagates in the two-dimensional free space. Loosely speaking, in concept, the elliptical reflector has advantages over the waveguide taper in terms of size because in the elliptical reflector, the light beam diffracts freely in free space which required minimum length to achieve prescribed beam size. Moreover, the reflected beam reuses the area occupied by the reflector, which minimizes the device size. Besides, propagation of light beam in the free space provides advantage over propagation of light beam in the waveguide in that it avoids the optical loss on the side wall of a waveguide due to the surface roughness in. This is especially significant in the nano-scale strongly guiding PICs. As for the case of waveguide crossing, different light beams do not affect each other when propagating in the free space. This enables us to use the elliptical reflector to realize high efficiency waveguide crossings. The waveguide crossing is a basic component in PICs. The waveguide crossing requires low transmission loss and low crosstalk to ensure the performance of PICs especially when large amount of the crossings are employed. Waveguide crossings structures with transmission loss less than 0.2 dB and crosstalk less than $-30$ dB are recently proposed [12-15]. The transmission loss is defined as the loss in power in the lowest-order beam mode in the output waveguide compared to the input waveguide in one channel of a waveguide crossing. The crosstalk is defined as the ratio of power of the light beam in the output waveguide in a non-working channel with respect to that in the working channel. The Direct Waveguide Crossing (DWC) is a straightforward and widely used type of waveguide crossing which directly crosses two channel waveguides. However, its crosstalk is high and its transmission efficiency drops quickly when waveguide width is smaller than 3 $\mu$m. We show that the Waveguide Crossing with Elliptical Reflectors (WCER) has significantly higher lowest-order mode transmission efficiency than the DWC and much less crosstalk ($\leq -30$ dB) even when the waveguide width is less than 3 $\mu$m [24].

4.1. Application of the Near Field design approach: device size minimization of compact spot size converter

The elliptical reflector can be used to connect two waveguides of different widths, similar to a waveguide taper. Here we discuss the case that the waveguide width is enlarged. In such case, the input and output waveguide widths, and in turn the incident and reflected Gaussian beam waist radii $w_{in}, w_{out}$ are prescribed and we have $\chi > 1$. If we were to minimize the dimension of the device size, we should choose $L_{out}$ and $L_{in}$ such that the sum of the two is minimal. The contour of the sum of $L_{in}$ and $L_{out}$ is illustrated in Fig. 5 in dotted lines, the closer to the origin the smaller the sum.

4.2. Application of the Far Field design approach: device size minimization of compact waveguide crossing

Elliptical reflectors can also be used to form a waveguide crossing. As opposed to the SSC, WCER, a Near Field design for the reflected beam may not be a good choice. Instead, we need the reflected beam to be focused, in other words, the reflector should be in the Far Field of the reflected beam, so as to minimize the device size.

A schematic diagram of a WCER consisted of two identical channels is shown in Fig. 6a. Each of the channels includes one input waveguides and one output waveguide, which have the same widths. To construct one of the channels, we use two identical elliptical reflectors in series. The second reflector is placed such that it is symmetric to the first reflector with respect to the center of the beam waist of the reflected beam of the first reflector. Therefore, the field profile on the second reflector will be same as that on the first reflector in terms of the lowest order
Here, we only show that given the corresponding to waveguide width and refractive index of the waveguide core is $DWG_{in}$, refractive index of the waveguide cladding is $n_{eclad}$, beam at wavelength 1.55 μm, beam size be $w_{win}$ and $w_{wout}$, how to determine $L_{in}$, $R_{in}$, $R_{out}$. The example is described as follows: a TE polarized beam at wavelength 1.55 μm is set as input beam. The effective refractive index of the waveguide core is $n_{core}$=3.2, the effective refractive index of cladding is $n_{eclad}$=1. Let the incident Gaussian beam waist radius be $w_{in}$=0.219 μm (corresponding to waveguide width $DWG_{in}$=0.7 μm), the output Gaussian beam size be $w_{out}$=0.546 μm (corresponding to waveguide width $DWG_{out}$=1.55 μm), and the turning angle $θ=90°$.

Step 1: Choose $β_{L,in}$ and $β_{L,out}$. Since $\chi=w_{out}/w_{in}>\sqrt{2}$, from Fig. 5 we know that the incident beam can only be the Far Field design. As a Far Field design, we choose $β_{L,in}=32$ since it gives $β_{L,out}=13$, which is a Far Field design.

Step 2: Determine $L_{in}$, $R_{in}$ and $L_{out}$, $R_{out}$. For the incident beam, from the definition of $β_{L,in}$, we have $L_{in}=β_{L,in}Z_{R,in}=9.99 μm$. Using (6), this gives $R_{in}=L_{in}+\sqrt{Z_{R,in}L_{in}}=10 μm$. Similarly, for the reflected beam, we have $L_{out}=β_{L,out}Z_{R,out}=24.85 μm$. This gives $R_{out}=L_{out}+\sqrt{Z_{R,out}L_{out}}=25 μm$.

Step 3: Determine the values of $a$ and $b$. Following (12)–(14), the parameters of the elliptical reflector can be calculated as

$$a=(R_{in}+R_{out})/2=17.50 μm \quad \text{and} \quad c=\sqrt{R_{in}^2+R_{out}^2-2R_{in}R_{out}\cos θ}=13.47 μm, \quad b=\sqrt{a^2-c^2}=11.18 μm.$$

Note that the reflector needs not to be metal coated. This is because for diffracted beam from a 0.7 μm input waveguide, this reflector provides Total Internal Reflection even at the edges of the reflector. Fig. 7 shows the FDTD simulation of above design. The left figure shows the two-dimensional layout. The right figure shows the $H_y$ snapshot. Then we mirror the reflector and the input waveguide with respect to the beam waist of the reflected beam, which is at $Z=L_{out}=24.85 μm$. The resultant reflector becomes the second reflector in a channel in a waveguide crossing. The resultant waveguide becomes the output waveguide in this channel. Then by duplicating and rotating this channel, the second channel is formed. The waveguide crossing is thus constructed as shown in Fig. 6c and d. Fig. 6c shows the two-dimensional layout.
The dependence of the lowest order mode transmission efficiency and crosstalk on wavelength for waveguide crossings using elliptical reflector (WCER) is below 1625 nm bands in telecom windows. It is also observed that the efficiency and crosstalk. In addition, compared to other crossing structures as mentioned in the Introduction section such as WCER outperforms the DWC both in terms of transmission efficiency of 87% and crosstalk of 17 dB. Therefore WCER outperforms the DWC both in terms of transmission efficiency and crosstalk. In addition, compared to other crossing structures as mentioned in the Introduction section such as waveguide crossings using MMI or photonic crystal structures, WCER have the advantage in that it is less wavelength sensitive. This is shown in Fig. 8. In Fig. 8 it can be seen that the lowest order mode transmission efficiency of WCER changes by less than 2% as the wavelength varies across the C and L (1530 nm – 1625 nm) bands in telecom windows. It is also observed that the crosstalk is below –40 dB across this wavelength window.

5. Conclusion

We introduced a two-dimensional Gaussian beam mode analysis to formulate the transformation of light beams in an elliptical reflector. The Gaussian beam formulation enables us to determine the shape and placement of the elliptical reflector, as well as the reflected beam physical parameters, such as beam waist size and beam waist position. We pointed out that depending on the design parameters the incident and reflected beams fall into Near field or Far Field scenarios. Because the reflected beams of the two scenarios entail different spatial configuration of devices in which elliptical reflectors are used, we introduce a methodology that chooses Near Field scenario design or Far Field scenario design to minimize the footprint of such devices. This is demonstrated by a waveguide crossing designed in such a way, which outperforms the conventional direct waveguide crossing in terms of the lowest-order mode transmission efficiency and the crosstalk.

Appendix

Here we derive the phase advancement of Gaussian beam in 2D planar waveguide as follows. First, we show how the phase front progresses in Gaussian beam. Using (4), not considering the Gouy phase and initial phase terms, the Gaussian modes share phase terms

$$-j k z - j k x^2 / (2 R(z))$$

In comparison, the phase term of a circular wave should be like

$$-j k z - j k x^2 / (2 z)$$

(31)

Recall that the reflector is designed such that at C, $R_g$ is transformed to $R_{out}$. In other words, the reflector is capable of transforming the phase fronts of incident and reflected circular beams centered at $O_1$ and $O_2$, respectively (Geometric optics). Let $Z = z - z_0 = R_g(z)$. The subscript $g$ indicates it is geometric optics. At C, $R_g$ should equal $R_{out}$ for the incident beam and $R_{out}$ for the reflected beam. The phase front of a Gaussian beam does not progress like a circular beam. The rate of progress like a circular beam. The rate of distortions are negligible to the first order[22].

We introduced a two-dimensional Gaussian beam mode analysis to formulate the transformation of light beams in an elliptical reflector. The Gaussian beam formulation enables us to determine the shape and placement of the elliptical reflector, as well as the reflected beam physical parameters, such as beam waist size and beam waist position. We pointed out that depending on the design parameters the incident and reflected beams fall into Near field or Far Field scenarios. Because the reflected beams of the two scenarios entail different spatial configuration of devices in which elliptical reflectors are used, we introduce a methodology that chooses Near Field scenario design or Far Field scenario design to minimize the footprint of such devices. This is demonstrated by a waveguide crossing designed in such a way, which outperforms the conventional direct waveguide crossing in terms of the lowest-order mode transmission efficiency and the crosstalk.


[25] Note that a more accurate mode matching can be done by using mode solver to solve for the 3D mode width exactly for the actual channel waveguide and then use that to match to the 2D Gaussian mode. This will be more accurate than using the channel waveguide mode width from 2D effective index solution. Typically, effective index method mode approximation is good enough for channel waveguide width of down to about 1 μm in the case of semiconductor waveguide at 1500 nm. Below that, better accuracy could be obtained by 3D mode solver. Once the mode width at the waveguide mouth is matched well, the rest of the diffraction computations in the planar waveguiding region shall be adequate for the design applications described in this paper.